

9/19/17 • FOILING

VS

DISTRIBUTING

$$(5x+3)(x+4) = 5x^2 + 20x + 3x + 12$$

$$(x+1)(x^2+3x+4) = x^3 + 3x^2 + 4x$$

$$x^2 + 3x + 4 \\ = x^3 + 4x^2 + 7x + 4$$

$$3(x+1) = 3x+3$$

$$\star (2x+1)^2 = (2x+1)(2x+1)$$

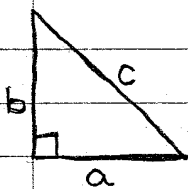
$$4x^2 + 1 \neq 4x^2 + 4x + 1$$

$$(AB)^2 = ABAB$$

$$A^2B^2$$

SPECIAL TRIANGLES

$$a^2 + b^2 = c^2 \quad \angle = 90^\circ \text{ Right triangle}$$

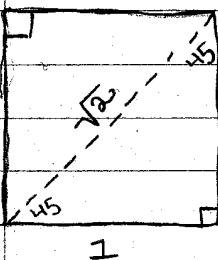
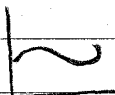


$$3^2 + 4^2 = 9 + 16 = 25 = 5^2$$

Pythagorean Triples

(5, 12, 13)

TRIG:



$$90^\circ + 2\alpha = 180$$

$$2\alpha = 90$$

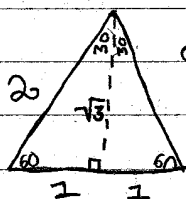
$$\alpha = 45^\circ$$

ex) $2, \sqrt{2}, \sqrt{2}$

↳ is a 45/45/90 Δ because

Pythagorean theorem works,

+ Common multiple = $\sqrt{2}$



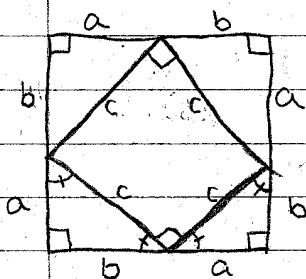
$$c^2 = a^2 + b^2$$

$$4 = 1 + b^2$$

$$3 = b^2$$

ex) 3, 4, 5

P. I. PROOF:



$$\text{Area of Big } \square = (a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$$

$$\text{Area of 4 } \Delta s = 4(\frac{1}{2} ab)$$

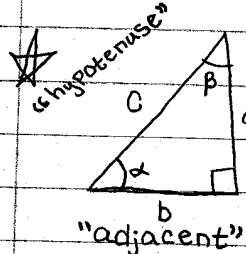
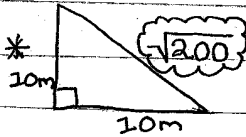
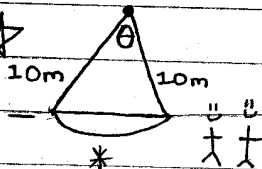
$$\text{Area of Small } \square = c^2$$

$$a^2 + 2ab + b^2 = 2ab + c^2$$

$$a^2 + b^2 = c^2$$

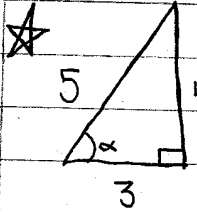


9/20/17 →

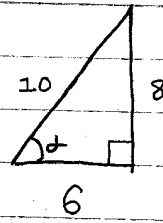


$$\begin{cases} \sin(\alpha) = \frac{a}{c} \rightarrow \text{opp/hyp} \\ \cos(\alpha) = \frac{b}{c} \rightarrow \text{adj/hyp} \\ \cos(\beta) = \sin(\alpha) \end{cases}$$

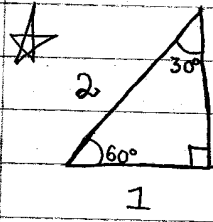
$$\begin{cases} \tan(\alpha) = \frac{a}{b} \rightarrow \text{opp/adj} \\ \tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)} \\ \frac{a/c}{b/c} = \frac{a}{b} \rightarrow \text{tangent} \end{cases}$$



$$\begin{aligned} \sin(\alpha) &= \frac{4}{5} \\ \cos(\alpha) &= \frac{3}{5} \end{aligned}$$

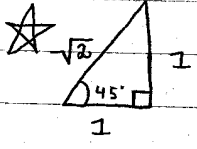


$$\sin(\alpha) = \frac{8}{10} = \frac{4}{5}$$



$$\begin{cases} \sin(60) = \frac{\sqrt{3}}{2} \\ \sin(30) = \frac{1}{2} \\ \cos(60) = \frac{1}{2} \\ \cos(30) = \frac{\sqrt{3}}{2} \end{cases}$$

$$\begin{aligned} \star (\sin(\alpha))^2 + \cos^2(\alpha) &= 1 \\ \textcircled{1} \sin^2(60) &= \frac{3}{4} \\ \cos^2(60) &= \frac{1}{4} \end{aligned}$$

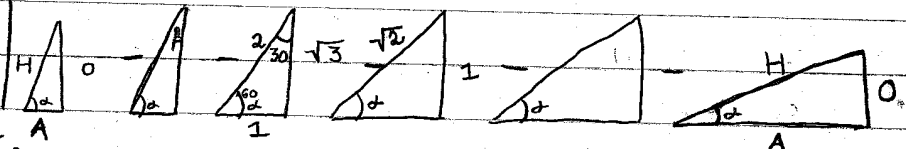


$$\begin{aligned} \sin^2(60) + \cos^2(60) &= 1 \\ \sin^2(45) &= \frac{2}{4} \\ \cos^2(45) &= \frac{2}{4} \\ \sin^2(\alpha) &= \frac{a^2}{c^2} \\ \cos^2(\alpha) &= \frac{b^2}{c^2} \\ \textcircled{2} \sin^2(\alpha) + \cos^2(\alpha) &= \frac{a^2}{c^2} + \frac{b^2}{c^2} \\ \frac{a^2 + b^2}{c^2} &= \frac{c^2}{c^2} = 1 \end{aligned}$$

$\star \alpha < 90^\circ \rightarrow$ acute
 $\alpha = 90^\circ \rightarrow$ right
 $\alpha > 90^\circ \rightarrow$ obtuse

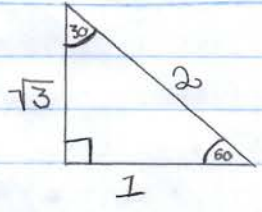
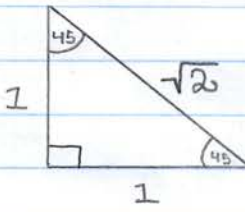
9/22/17 → SOH-CAH-TOA

$$\begin{aligned} \sin(90) &= 1 & \sin(0) &= 0 \\ \cos(90) &= 0 & \cos(0) &= 1 \end{aligned}$$



Hipparchus → Father of Trig.

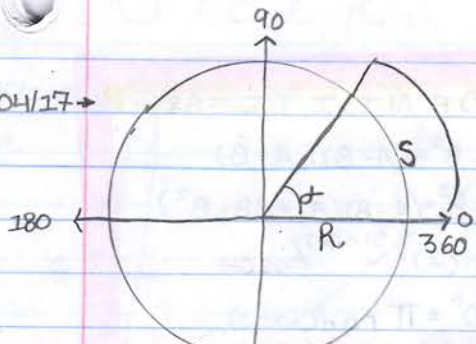
Cosines+ Sines of Special triangles

TRIANGLE	COSINE (^{adj} /hyp)	SINE (^{opp} /hyp)
	$\cos(30) = \frac{\sqrt{3}}{2}$ $\cos(60) = \frac{1}{2}$ $\cos(90) = 0$	$\sin(30) = \frac{1}{2}$ $\sin(60) = \frac{\sqrt{3}}{2}$ $\sin(90) = 1$
	$\cos(45) = \frac{1}{\sqrt{2}}$ $\cos(90) = 0$	$\sin(45) = \frac{1}{\sqrt{2}}$ $\sin(90) = 1$
<p>it is not possible to draw a triangle with a zero degree angle measure.</p>	$\cos(0) = 1$	$\sin(0) = 0$

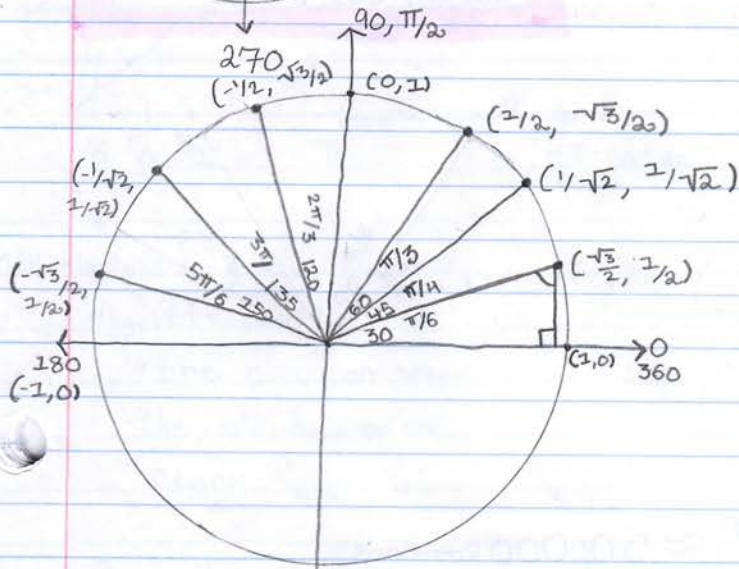
9/30/17 →

Sin	Cos	tan	Sec	cot	csc
$\frac{o}{h}$	$\frac{a}{h}$	$\frac{o}{a}$	$\frac{h}{a}$	$\frac{a}{o}$	$\frac{h}{o}$
$\frac{\text{leg}}{\text{hypo}}$	$\frac{\text{leg}}{\text{hypo}}$	$\frac{\text{leg}}{\text{leg}}$	$\frac{\text{hypo}}{\text{leg}}$	$\frac{\text{leg}}{\text{leg}}$	$\frac{\text{hypo}}{\text{leg}}$
			$\frac{1}{\cos}$	$\frac{1}{\tan}$	$\frac{1}{\sin}$
SINE	COSINE	TANGENT	SECANT	COTANGENT	COSECANT

10/04/17

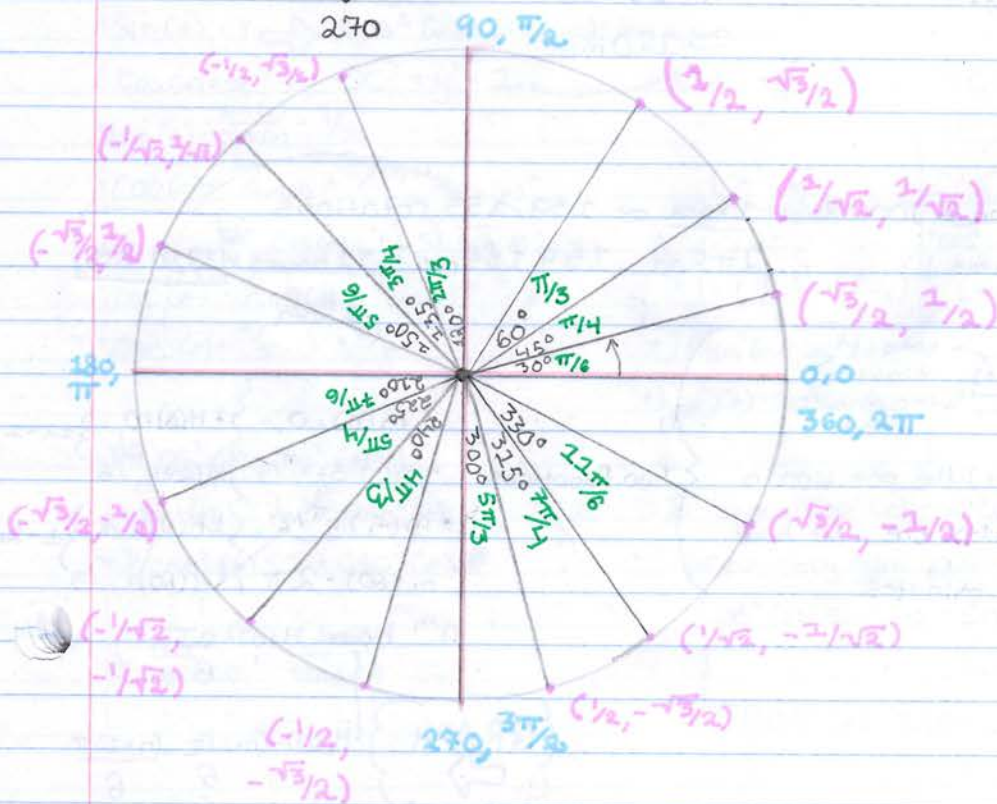


$\theta = S/R$
 $180^\circ = \pi \text{ Rad}$
 $\frac{180}{\pi} \text{ degrees} = 1 \text{ Radian}$
 $7 \text{ radian} \cdot \frac{180}{\pi} = \frac{1260}{\pi} = 401.07$



$$\frac{120}{1} \cdot \frac{\pi}{180} = \frac{2}{3}\pi$$

$$\frac{130\pi}{180} = \frac{5\pi}{6}$$



10/11/17 Homework Question:

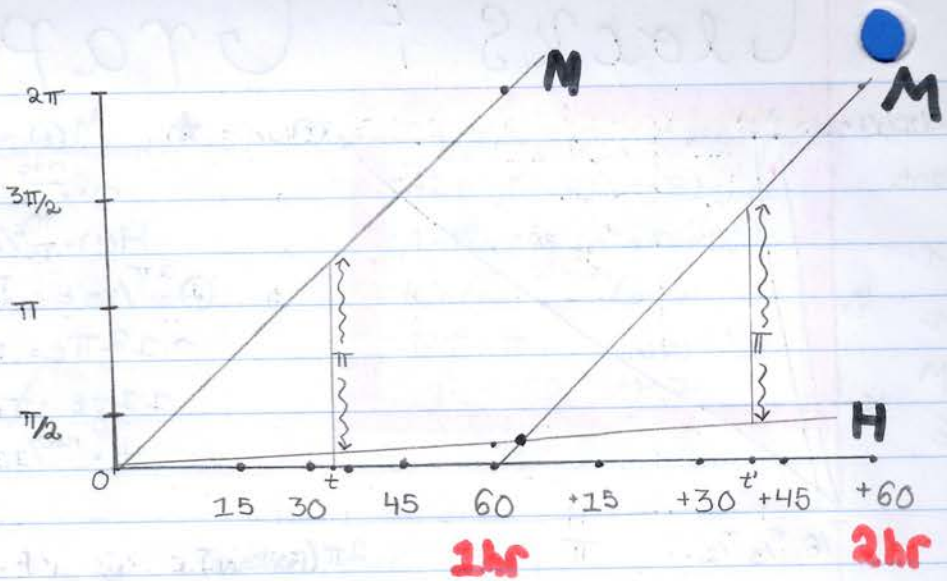
$$\frac{2\pi}{60} = \frac{\pi t}{360} + \pi$$

$$\frac{360 \times 2}{60} t = t + 360$$

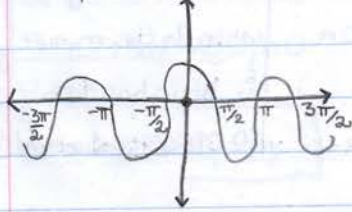
$$12t = t + 360$$

$$11t = 360$$

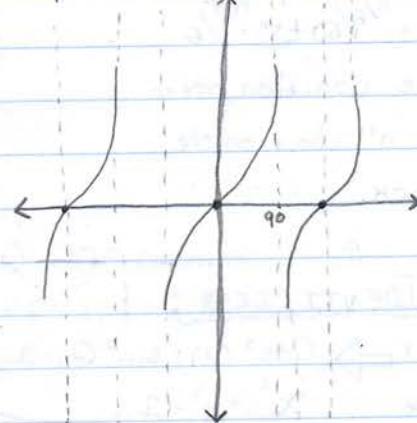
$$t = \frac{360}{11} \approx 32.7 \text{ minutes}$$



10/12/17 Graph of cos(x)



Graph of tan(x)



10/13/17 H(t) = \frac{\pi}{360}t + \frac{n\pi}{6}

$$M(t) = \frac{2\pi}{60}t$$

So, when will H(t) - M(t) = \pi?

$$\frac{\pi}{360}t + \frac{n\pi}{6} - \frac{2\pi}{60}t = \pi$$

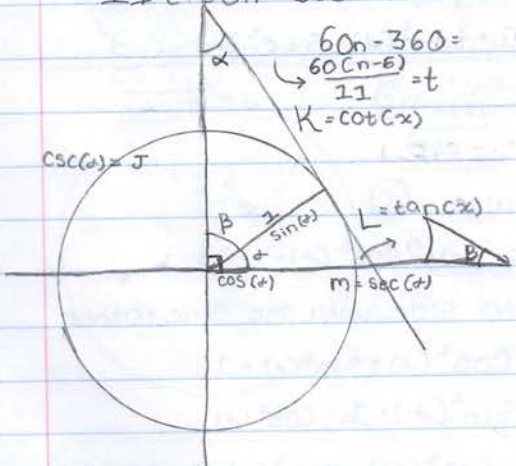
$$11t + 60n - 12t = 360$$

$$-11t + 60n = 360$$

$$60n - 360 =$$

$$\frac{60(n-6)}{11} = t$$

$$K = \cot(x)$$



$$\tan(\alpha) = L$$

$$\cos(\alpha) = \frac{1}{m}, m = \sec(\alpha)$$

$$\tan(\alpha) = \frac{1}{K}, K = \cot(\alpha)$$

$$\sin(\alpha) = \frac{1}{J}, J = \csc(\alpha)$$

$$\cos^2 \alpha + \sin^2 \alpha = 1$$

$$1 + L^2 = m^2$$

$$1 + \tan^2 \alpha = \sec^2 \alpha$$

$$1 + \cot^2 \alpha = \csc^2 \alpha$$

Functions

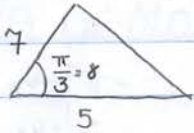
11/29/17 → DOMAIN → FUNCTION → RANGE

x-values

y-values

*and input ≠ ratio.
↳ output

• Review sheet #3 ✓

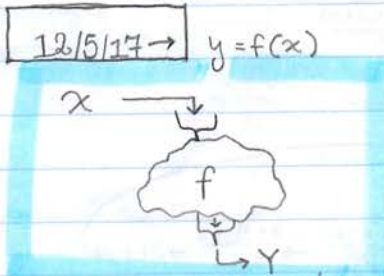


$AC(S, A, S) = \frac{1}{2} \cdot a \cdot b \cdot \sin \alpha$
 ↳ triangle congruence
 ↳ inputs

• functions need to be "well-defined"
↳ For every input, there is exactly one output.

12/1/17 → Euler → FUNCTIONS

- Switzerland → $e^{i\pi} + 1 = 0$
- 1707 - 1783
- pastor? 5/13
- St. Petersburg
- Berlin academy
- Fred II → cyclops _{↳ blind}
- Most prolific
- Brain hemorrhage → death



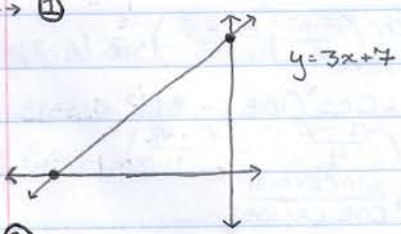
DOMAIN	RANGE	DOMAIN	RANGE	DOMAIN	RANGE
1	4	1	4	1	4
2	5	2	5	2	5
3	6	3	6	3	6
	7		7		7

$f(1)=4, f(2)=4$ g h

* 2 OUTPUTS

↳ NOT FUNCTION

12/5/17 → ①



FUNCTION: every input has exactly one output.

Vertical line test

②

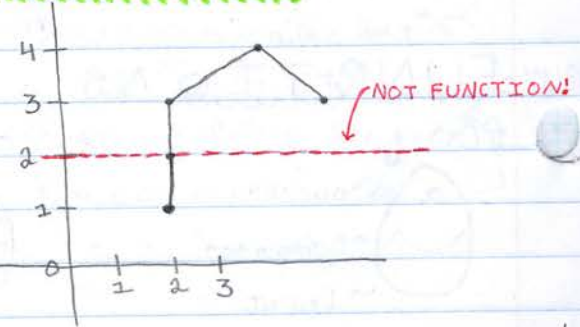
$y = ax^2 + bx + c$

$y = 2(x^2) + 5x + 3$

x-coordinate vertex →

$x = \frac{-b}{2a} = \frac{-5}{4} \rightarrow y = a\left(\frac{-b}{2a}\right)^2 - \frac{b^2}{2a} + c$

$2\left(\frac{-5}{4}\right)^2 - \frac{25}{4} + 3$ $y = \frac{b^2}{4a} - \frac{2b^2}{4a} + \frac{4ac}{4a} =$
 $+\frac{25}{8} - \frac{50}{8} + \frac{24}{8} = -\frac{7}{8}$ $-\frac{b^2 + 4ac}{4a}$ ↳ of vertex



↳

Functions \rightarrow EVEN or ODD

Lucy
12/22/17

	x	x^2	x^3	x^4	x^5	x^6	x^{odd}	x^{even}
-2	-2	4	-8	16	-32	64	-2 ^{odd}	2 ^{even}
-1	-1	1	-1	1	-1	1	-1	1
0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1
2	2	4	8	16	32	64	2 ^{odd}	2 ^{even}

even function: $f(-x) = f(x)$
 odd function: $f(-x) = -f(x)$
 $f(x) = x^3 \rightarrow f(-1) = (-1)^3 = -1$
 $\rightarrow f(1) = 1^3 = 1$

* $\cos(x)$ is even
 $\sin(x)$ is odd
 $\tan(x)$ is odd

* $\cos(-x) = \cos(x)$
 $\cos(-45) = \frac{1}{\sqrt{2}}$
 $\sin(-45) = -\frac{1}{\sqrt{2}}$
 $\tan(x)$ is odd
 $\tan(-x) = \frac{\sin(-x)}{\cos(-x)}$
 $= \frac{-\sin(x)}{\cos(x)} = -\tan(x)$

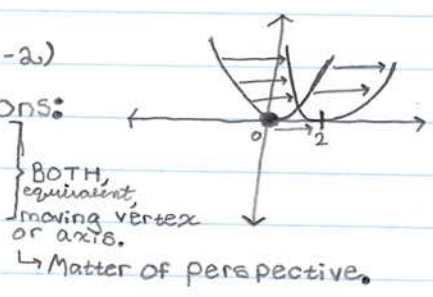
$f(x) = \frac{x^3 - 5x}{\sqrt{1+x^2}}$ $\xrightarrow{\text{PLUG IN } (-x)}$ $f(-x) = \frac{(-x)^3 - 5(-x)}{\sqrt{1+(-x)^2}}$ $\rightarrow \frac{-x^3 + 5x}{\sqrt{1+x^2}}$
 \rightarrow factor out a negative: $-\frac{(x^3 - 5x)}{\sqrt{1+x^2}}$

$f(-x) = -f(x) \rightarrow$ ODD
 $= f(x) \rightarrow$ EVEN
 $= ? \rightarrow$ NEITHER

$f(x) = x^2$; $g(x) = (x-2)^2 \rightarrow g(x) = f(x-2)$

Possible Transformation Descriptions:

- ① graph move 2 to the right
- ② Domain 2 to left



$f(x) = x^2$; $g(x) = f(2x) \rightarrow g(x) = (2x)^2$

- ① Domain narrows by a factor of 2.
- ② Graph.

